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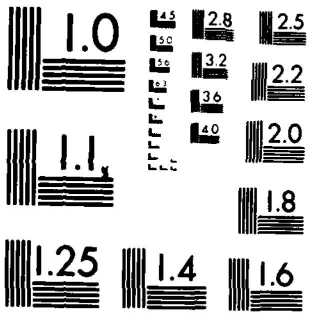

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ALGORITHMS FOR NONLINEAR PROGRAMMING

FINAL REPORT 83-1

BY

DONALD GOLDFARB

JULY 1983

U. S. ARMY RESEARCH OFFICE

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Several algorithms in linear, quadratic, and nonlinear programming have been developed and analyzed. These include:

(i) The development of relaxation methods for finding a feasible solution to a system of linear inequalities based upon generating surrogate constraints.

(ii) The worst-case behaviour of the shadow-vertex simplex algoritm was shown to be the exponential.

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(iii) Necessary and sufficient conditions for versions of iterative methods, including the Jacobi, Gauss-Seidel and SOR methods, designed for solving equality constrained quadratic programs have been obtained.

(iv) The development of numerically stable and efficient implementations of primal methods for quadratic programming.

(v) The development of optimal algorithms for estimating Jacobian and Hessian matrices arising in finite difference calculations.

A. PROBLEM STATEMENT

Our research under this grant has focused on the development and analysis of algorithms for mathematical programming. The algorithms that were developed and studied can be categorized according to the areas of linear, quadratic, and nonlinear programming.

In linear programming a number of problems were studied including (i) relaxation methods for finding a feasible solution to a system of linear inequalities, (ii) the computational complexity of the "shadow-vertex" simplex algorithm, and (iii) methods for solving the assignment, or maximum weighted matching problem.

In quadratic programming we studied (i) iterative methods for solving equality constrained quadratic programs and (ii) how to implement primal methods in a numerically stable and efficient way.

In the area of nonlinear programming we considered the problem of estimating Hessian and Jacobian matrices with a minimal number of gradient and function differences in certain special cases.

B. SUMMARY OF RESEARCH RESULTS

1. Linear Programming Algorithms

(i) Several relaxation methods have been developed for finding a feasible point for a system of linear inequalities. These methods are based upon generating "surrogate" linear inequalities, a technique developed in our earlier work on the ellipsoid method. Preliminary computational testing of these methods has been performed and some convergence results have been obtained. Further testing and theoretical analysis is planned.

(ii) There is currently a great deal of interest in the computational complexity of algorithms, both in the sense of worst-case behavior and average-case behavior. Because of its extensive use, the computational complexity of the simplex method is of particular importance. It has been known for some time that some of the standard versions of the simplex method have exponential worst-case time bounds. (This was proved for the steepest edge simplex method by the principal investigator.) Recently, a variant of the simplex algorithm, the "shadow-vertex algorithm" has been shown to be polynomial in both the number of the variables and constraints on the average. We have shown that in the worst case this variant can still require an exponential number of pivots.

(iii) Work has begun on the development of an efficient dual simplex algorithm for solving the assignment problem.

2. Quadratic Programming Algorithms

(i) We have obtained necessary and sufficient conditions for the convergence of a class of iterative methods for solving equality constrained quadratic programs. These methods include generalizations of the Jacobi, Gauss-Seidel and SOR methods. Our current goal is to study how to use these methods within successive quadratic programming algorithms for solving large nonlinear programming problems.

2. (Cont...)

(ii) Our work on the development of efficient and numerically stable methods for solving quadratic programming problems has continued. Based on our earlier work with Dr. A. Idnani on dual and primal-dual quadratic programming methods we have developed efficient and numerically stable implementations for purely primal methods.

3. *Nonlinear Programming Algorithms*

(i) Methods for estimating Jacobian and Hessian matrices using a minimal number of function and gradient differences have been developed for certain special cases. The cases studied were finite difference approximations to partial differential equations. The optimal schemes that were developed were based upon the "computational molecule" or "stencil" of the underlying finite difference operator. A "tearing" approach which is applicable to more complicated structures was also developed.

C. PUBLICATIONS

D. Goldfarb and Ph.L. Toint "Optimal Estimation of Jacobian and Hessian Matrices that Arise in Finite Difference Calculations", submitted to Mathematics of Computation.

D. Goldfarb "Worst-Case Complexity of the Shadow Vertex Simplex Algorithm" Columbia University IE&OR Technical Report (to be submitted to a journal)

D. PARTICIPANTING SCIENTIFIC PERSONNEL

Principal Investigator: Professor Donald Goldfarb

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